## Properties of Inequality
Let $a$, $b$, and $c$ represent real numbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Statement</th>
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<tbody>
<tr>
<td>Transitive Property</td>
<td>If $a \leq b$ and $b \leq c$, then $a \leq c$.</td>
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<tr>
<td>Addition Property</td>
<td>If $a \leq b$, then $a + c \leq b + c$.</td>
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<tr>
<td>Subtraction Property</td>
<td>If $a \leq b$, then $a - c \leq b - c$.</td>
</tr>
<tr>
<td>Multiplication Property</td>
<td>If $a \leq b$ and $c &gt; 0$ (c is positive), then $ac \leq bc$.</td>
</tr>
<tr>
<td></td>
<td>If $a \leq b$ and $c &lt; 0$ (c is negative), then $ac \geq bc$.</td>
</tr>
<tr>
<td>Division Property</td>
<td>If $a \leq b$ and $c &gt; 0$ (c is positive), then $\frac{a}{c} \leq \frac{b}{c}$.</td>
</tr>
<tr>
<td></td>
<td>If $a \leq b$ and $c &lt; 0$ (c is negative), then $\frac{a}{c} \geq \frac{b}{c}$.</td>
</tr>
</tbody>
</table>

Key Idea: When you multiply or divide by a negative, you must flip the inequality sign.

### Example: Solving and Graphing Inequalities

1. Solve each inequality. Graph the solution.

   **a.** $3x - 12 < 3$
   
   \[
   \begin{align*}
   3x & < 15 \\
   x & < 5
   \end{align*}
   \]

   **b.** $6 + \left(\frac{2}{3} - x\right) \leq 41$
   
   \[
   \begin{align*}
   \frac{12}{3} - x & \leq 41 \\
   \frac{12}{3} - 10 & \leq 41 \\
   -8 & \leq 41 \\
   -\frac{8}{5} & \leq 41 \\
   x & \geq -5
   \end{align*}
   \]

1. Solve each inequality. Graph the solution.

   **a.** $3x - 6 < 27$
   
   \[
   \begin{align*}
   3x & < 33 \\
   x & < 11
   \end{align*}
   \]

   **b.** $12 \geq \left(\frac{3n + 1}{6}\right) + 22$
   
   \[
   \begin{align*}
   12 & \geq \frac{3n + 1}{6} + 22 \\
   12 & \geq \frac{3n + 23}{6} \\
   -24 & \geq 3n + 23 \\
   -12 & \geq 6n \\
   -2 & \geq n
   \end{align*}
   \]

### Diagram

- Graph showing the solution for each inequality with number lines and arrows indicating the solution intervals.
Some inequalities have no solution, and some are true for all real numbers. If your final answer is \( a \leq b \) where \( a \) and \( b \) are real numbers and it is true, the solution is all real numbers. If it is false, there is NO solution.

**Example:**

**No Solutions or All Real Numbers as Solutions**

Solve each inequality. Graph the solution.

- **a.** \( 2x - 3 > 2(x - 5) \)
  
  \[
  \begin{align*}
  2x - 3 &> 2x - 10 \\
  -3 &> -10 \\
  0 &> -7 \quad \text{true statement}
  \end{align*}
  \]

  So all real numbers are solutions.

- **b.** \( 7x + 6 < 7(x - 4) \)
  
  \[
  \begin{align*}
  7x + 6 &< 7x - 28 \\
  -7x &< -28 \\
  -1 &< 4 \quad \text{false statement}
  \end{align*}
  \]

  So there are no solutions.

**2.**

**a.** Solve \( 2x < 3(x + 1) + 3 \). Graph the solution.

\[
\begin{align*}
2x &< 3x + 2 + 3 \\
2x &< 3x + 5 \\
-2x &< -2x \\
0 &< 5 \quad \text{true statement}
\end{align*}
\]

All real numbers are solutions.

**b.** Solve \( 3(x - 3) + 7 \geq 4x + 1 \). Graph the solution.

\[
\begin{align*}
4x - 12 + 7 &\geq 4x + 1 \\
4x - 5 &\geq 4x + 1 \\
4x &\geq 4x + 6 \\
0 &\geq 6 \quad \text{false statement}
\end{align*}
\]

so no solutions.

**c. Critical Thinking** If possible, find values of \( a \) such that \( 2x + a > 2x \) has no solution. Then find values of \( a \) such that all real numbers are solutions.

- when \( a \) is negative, there is no solution.
- when \( a \) is positive, all real numbers are solutions.
### Example

**Revenue** The band shown at the left agrees to play for $200 plus 25% of the ticket sales. Find the ticket sales needed for the band to receive at least $500.

**Relate** $200 + 25\%$ of ticket sales $\geq$ $500$

**Define** Let $x$ = ticket sales (in dollars).

**Write** $200 + 0.25x \geq 500$

\[
\begin{align*}
-200 & \quad -200 \\
\frac{25x}{0.25} & \geq \frac{300}{0.25} \\
\frac{25}{0.25} x & \geq 1200 \\
x & \geq 1200
\end{align*}
\]

The ticket sales must be greater than or equal to $1200$.

---

3 A salesperson earns a salary of $700 per month plus 2% of the sales. What must the sales be if the salesperson is to have a monthly income of at least $1800?

Let $s$ represent the sales.

\[
\begin{align*}
700 + 0.02s & \geq 1800 \\
-700 & \quad -700 \\
0.02s & \geq 1100 \\
\frac{0.02}{0.02} s & \geq \frac{1100}{0.02} \\
s & \geq 5500
\end{align*}
\]

At least $5500$

---

### Compound Inequalities

**Compound Inequality:** A pair of inequalities joined by "and" or "or".

To solve a compound inequality containing "and" find all the values of the variables that make BOTH inequalities true.

To solve a compound inequality containing "or" find all the values of the variables that make AT LEAST ONE of the inequalities true.
**Example 4** Compound Inequality Containing And

Graph the solution of $3x - 1 > -28$ and $2x + 7 < 19$.

\[
\begin{align*}
3x - 1 & > -28 \\
+1 & +1 \\
3x & > -27 \\
\frac{3x}{3} & \leq \frac{-27}{3} \\
x & > -9
\end{align*}
\]

\[
\begin{align*}
2x + 7 & < 19 \\
\frac{2x}{2} & < \frac{12}{2} \\
x & < 6
\end{align*}
\]

This compound inequality can be written as $-9 < x < 6$.

**Example 4**

Graph the solution of $2x > x + 6$ and $x - 7 < 2$.

\[
\begin{align*}
2x & > x + 6 \\
-2x & -x \\
x & > 6
\end{align*}
\]

\[
\begin{align*}
x - 7 & < 2 \\
+7 & +7 \\
x & < 9
\end{align*}
\]

$6 < x < 9$

**Example 5** Compound Inequality Containing Or

Graph the solution of $4y - 2 \geq 14$ or $3y - 4 \leq -13$.

\[
\begin{align*}
4y - 2 & \geq 14 \\
+2 & +2 \\
4y & \geq 16 \\
\frac{4y}{4} & \geq \frac{16}{4} \\
y & \geq 4
\end{align*}
\]

\[
\begin{align*}
3y - 4 & \leq -13 \\
+4 & +4 \\
3y & \leq -9 \\
\frac{3y}{3} & \leq \frac{-9}{3} \\
y & \leq -3
\end{align*}
\]

$y \geq 4$ or $y \leq -3$
5 Solve the compound inequality \( x - 1 < 3 \) or \( x + 3 > 8 \). Graph the solution.

\[
\begin{align*}
\frac{x - 1}{1} < 3 & \quad \text{or} \quad \frac{x + 3}{1} > 8 \\
\underline{\text{or}} \\
\frac{x}{1} < 4 & \quad \text{or} \quad x > 5 \\
\end{align*}
\]

6 **Example**

**Real-World Connection**

**Multiple Choice** The ideal length of a bolt is 13.48 cm. The length can vary from the ideal by at most 0.03 cm. A machinist finds one bolt that is 13.67 cm long. By how much should the machinist decrease the length so the bolt can be used?

- (A) between 13.45 cm and 13.51 cm
- (B) between 13.64 cm and 13.70 cm
- (C) between 0.16 cm and 0.22 cm
- (D) between 0.13 cm and 0.19 cm

**Relate** minimum length ≤ final length ≤ maximum length

**Define** Let \( x \) = number of centimeters to remove.

**Write** \( 13.48 - 0.03 \leq 13.67 - x \leq 13.48 + 0.03 \)

\[
\begin{align*}
13.45 & \leq 13.67 - x \leq 13.51 \\
-13.67 & \leq -x \leq -13.51 \\
\frac{-0.22}{-1} & \geq x \geq \frac{-0.16}{-1} \\
0.22 & \geq x \geq 0.16
\end{align*}
\]

The machinist must remove at least 0.16 cm and no more than 0.22 cm.